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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 615

A STUDY OF CURVILINEAR FLIGHT

By Helmuth Kruse

Zeitschrift für Flugtechnik und Motorluftschiffahrt
January 28, 1931, Vol. 22, No. 2
Verlag von R. Oldenbourg, München und Berlin

Washington
April, 1931

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By Helmuth Kruse

N o t a t i o n

α° = angle of attack,

$\alpha_o{}^\circ$ = angle of attack for zero lift,

β° = angle of aileron setting,

c_a = lift coefficient,

c_n = coefficient of normal force,

c_w = drag coefficient,

c_{m_r} = coefficient of rolling moment,

$\frac{dc_a}{d\alpha}$ $1/^\circ$ = lift gradient,

$\frac{dc_n}{d\alpha}$ $1/^\circ$ = gradient of normal force,

d/t - = profile thickness,

b (m) = span,

t (m) = chord,

h (m) = gap,

l (m) = length of one aileron,

F (m^2) = wing area,

F_Q(m^2) = aileron area,

I (mkg s^2) = inertia moment about vertical axis.

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*"Untersuchung des Kurvenfluges," from Zeitschrift für Flugtechnik und Motorluftschiffahrt, Jan. 28, 1931, Vol. 22, No. 2, Verlag von R. Oldenbourg, München und Berlin.

M_r (mkg)	=	rolling moment due to aileron setting,
D (mkg)	=	damping moment,
μ (°)	=	inclination (bank),
v (m/s)	=	wind velocity,
q (kg/m²)	=	dynamic pressure,
ω (1/s)	=	angular speed of rotation,
$\frac{d\omega}{dT}$ (1/s²)	=	angular speed of acceleration,
T (s)	=	time

When an airplane describes a curve it takes a certain time for it to turn from level to inclined position and then back to level again. In the following, we express the motion about the horizontal axis as "roll" or "bank" and the motion perpendicular to the vertical axis, i.e., the actual curve, as "turn."

The moment of ~~wall~~ roll is

$$\underline{M_r} = c_{m_r}\ b^2\ t\ q = I\ \frac{d\omega}{dt} - D, \tag{1}$$

with coefficient of rolling moment c_{m_r} dependent on aileron setting β. Therefrom follows the time required to attain a certain inclined position by given angular velocity* as

$$T_R = \frac{12\ I}{\frac{\gamma}{2g}\ v\ t\ b^3\ \frac{dc_n}{d\alpha}}\ \ln \frac{1}{1 - \frac{\omega}{vk}\ \frac{dc_n}{d\alpha}} \tag{2}$$

with $k = \frac{12\ c_{m_r}}{b}$.

*Lachmann, "The Span as Underlying Basis in Airplane Design," Zeitschrift fur Flugtechnik und Motorluftschiffahrt, 1928, pp. 198-208.

Lachmann then assigns to each inclined position produced by a rotation at certain angular velocity a portion of the turn in the horizontal plane, and deduces the time of turn through an angle of 90° and 180°, respectively. Hereby it is assumed that the inclined position has reached its maximum after a 90° turn. According to Lachmann, the time for a 180° turn is

$$T_W = \frac{2}{\omega} \text{ arc cos } e^{-\frac{1}{2}\pi\omega v : g} \tag{3}$$

With these two equations (1) and (2), we can now determine the speed and time of roll and the time of turn. But the method presents one difficulty when defining rolling moment $\underline{M}_r$, or its coefficient c_{m_r}. According to Lachmann, c_{m_r} is assumed or estimated from wind-tunnel data. But in estimating, we venture into a very dubious field. In order to arrive at an approximation of the actual figures, it would necessitate a great amount of experimental test data on wings of many different profiles with different aileron settings and "aileron area/total wing area" ratio, which we, however, lack.

So we shall endeavor to interpret c_{m_r} mathematically from Wieselsberger and Asano's report, "Determination of Aerodynamic Forces and Moments Induced by the Ailerons of a Wing," Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1928, pp. 289-293. Even then it calls for the aerodynamically effective angle of attack of the wing section with deflected aileron: $\alpha + \alpha_Q$ and $\alpha - \alpha_Q$, respectively, which must be defined by calculation or

wind-tunnel experiments. Then the curve $c_a = f(\alpha)$ represents a series of almost parallel lines for a profile with different aileron settings (Fig. 1). So long as the lift gradient $\frac{dc_a}{d\alpha}$ is constant, they may be considered as parallel. At settings of $> \pm 15^\circ$ we encounter deviations which are attributable to the breakaway of the flow.

On the premises of various experiments, Toussaint* evaluates the lift of a profile with aileron setting at

$$c_a = A\ (\alpha_1 + \beta_0 + m\ \beta) \tag{4}$$

where

$$A = \frac{0.08\ (1 + \epsilon)\ \lambda}{\lambda + 1.57\ (1 + \epsilon)}$$

$$\lambda = \frac{\kappa^2\ b^2}{F},$$

κ^2 being Munk's value for biplane calculation slightly modified by Toussaint. Value ϵ is dependent only on the profile thickness, and Toussaint's formula $(1 + \epsilon) = \left(1 + 0.8\ \frac{d}{t}\right)$, is applicable only when $d/t < 0.13$. Factor A thus indicates the lift gradient. Toussaint's notation of the angles in equation (4) may be seen in Figure 2, with $(\alpha_1 + \beta_0)$ as angle for zero lift. A glance at Figure 1, which exhibits Wieselsberger's method, reveals that angle α_Q and Toussaint's $m\ \beta$ are identical.

According to Toussaint, the coefficient m is:

$$m = m_0 - 0.005\beta \tag{5}$$

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*Toussaint, "L'Aviation actuelle," 1928, Librairie Félix Alcan, Paris.

where
$$m_0 = \frac{4}{\pi}\sqrt{\sigma}\,(1 - 0.215\ \sigma) \tag{6}$$
$$\sigma = \frac{F_Q}{F}$$

F_Q being the sum of the aileron areas and F the total wing area inclusive of ailerons. Consequently,

$$\alpha_Q = m\ \beta = \beta\left[\frac{4}{\pi}\sqrt{\frac{F_Q}{F}}\left(1 - 0.215\ \frac{F_Q}{F}\right) - 0.005\ \beta\right] \tag{7}$$

Applied to the Heinkel HD 35 it yields:

$$F_Q = 3.5\ m^2,\quad F = 32.4\ m^2 \quad \text{and} \quad \frac{F_Q}{F} = 0.1082$$
$$\alpha_Q = \beta\ (0.409 - 0.005\ \beta).$$

Toussaint then adduces an empirical value for m_0 which is stated to be better suited to wings with cut-out section and less aileron area:

$$m_0 = 1.04\sqrt{\sigma} = 0.343 \quad \text{(in our example)} \tag{8}$$

where the amount for m_0 is now slightly below that revealed by equation (6). Lastly, α_Q becomes

$$\alpha_Q = \beta\ (0.343 - 0.005\ \beta).$$

With α_Q, the curve $c_a = f\ (\alpha + \alpha_Q)$ is also known, for we assumed the lift gradient to be constant for all light curves with aileron settings.

Wieselsberger specifies the rolling moment as

$$\underline{M}_R = q\ b^3\ \zeta\ \alpha_Q \tag{9}$$

where ζ is contingent on the ratio of twice the length of the ailerons to the span and on the nondimensional parameter

$p = \frac{2\,b}{c_1\;t}$. The ζ value can be determined from a set of curves* so that the rolling moment can now be calculated. The application of (1) to (9) yields the coefficient of rolling moment

$$c_{m_r} = \frac{b}{t}\;\zeta\;\alpha_Q \tag{10}$$

Example

With $b_o = 11.00$ m $t_o = 1.55$ m $l_o = 3.02$ m

$b_u = 9.85$ m $t_u = 1.55$ m $l_u = 2.44$ m

for the Heinkel HD 35 biplane, we have

$$\frac{2\,l}{b} = 5.22 \qquad c_1 = 2.42 \qquad p = 5.53,$$

and consequently, $\zeta = 0.08$. Hence,

$$c_{m_r} = 0.538\;\alpha_Q$$

$$\alpha_Q = \beta\;(0.343 - 0.005\;\beta)$$

$$k = 1.15\;c_{m_r}.$$

Table I embraces the data for coefficient of rolling moment c_{m_r} at various aileron settings and the auxiliary quantity k as graphed on Figure 3.

Now we interpret $\omega = f\;(T_R)$ from equation (2) and at the same time deduce the effect of the inertia moment about the longitudinal axis. To this end we extend the calculation to two different inertia moments but with identical wing dimensions. It is assumed that one inertia moment is 25% greater than the other. An estimate reveals $I_1 = 340$ and $I_2 = 425$ mkg/s². The aerodynamic coefficients concede the lift gradient and the

*S. Wieselsberger, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1928, p. 291, Figure 5.

normal force gradient at

$$\frac{dc_n}{d\alpha^o} = 0.0669 \quad \text{and} \quad \frac{dc_n}{d\widehat{\alpha}} = 4.0,$$

as illustrated in Figure 4, where c_a, c_n, and c_w are H. Müller's data (Luftfahrtforschung V, 1) on the Heinkel HD 35, plotted against α. The flight speed is assumed at $V = 108$ km/h, and the calculation is to be made for three aileron settings, namely, $\beta = 2^o$, 8^o, and 16^o.

<u>For $\beta = 2^o$</u>

Table I shows $k = 0.00719$, and equation (2) yields for I_1 and I_2:

$$T_{R_1} = 0.700 \log \frac{1}{1 - 18.6\ \omega} \quad \text{and} \quad T_{R_2} = 0.875 \log \frac{1}{1 - 18.6\ \omega} .$$

In the same manner we obtain:

<u>For $\beta = 8^o$</u>

$k = 0.0259$, and

$$T_{R_1} = 0.700 \log \frac{1}{1 - 5.15\ \omega} \quad \text{and} \quad T_{R_2} = 0.875 \log \frac{1}{1 - 5.15\ \omega} .$$

and

<u>For $\beta = 16^o$</u>

$k = 0.0455$, and

$$T_{R_1} = 0.700 \log \frac{1}{1 - 2.93\ \omega} \quad \text{and} \quad T_{R_2} = 0.875 \log \frac{1}{1 - 2.93\ \omega} .$$

TABLE I

β°	0.005β	m	α°_Q	$\hat{\alpha}_Q$	c_{m_r}	k
2	0.01	0.333	0.666	0.01162	0.00625	0.00719
4	0.02	0.323	1.292	0.02255	0.01213	0.01397
8	0.04	0.303	2.424	0.04225	0.02256	0.02594
12	0.06	0.283	3.396	0.0591	0.0318	0.0366
16	0.08	0.263	4.208	0.0734	0.0395	0.0455
20	0.1	0.243	4.86	0.0848	0.0456	0.0525
(Compare Figure 3)						

The evaluation is carried through in Tables II-IV, and graphically shown in Figure 5, where the effect of the inertia moment is plotted against the angle and the speed of roll. Now it is apparent that $\omega = f\ (T_R)$ is asymptotic in its course toward $T = \infty$, as may be deduced from equation (2); because at maximum angular velocity we obtain $\log \frac{1}{0}$, that is, $T_R = \infty$. Accordingly, ω must become practically constant within a certain interval, and the calculation revealed this time to be the same for all three aileron settings, namely, $T_R = 1.61$ sec. Here it is assumed that the angular velocity is uniform when it reaches 99.5% of its maximum.

TABLE II

ω	ω°/s	18.6ω	$1-18.6\omega$	$\frac{1}{1-18.6\omega}$	log	T_1	μ_1	T_2	μ_2
0.02	1.148	0.372	0.628	1.595	0.2027	0.142	0.163	0.177	0.203
0.04	2.295	0.744	0.256	3.91	0.5921	0.414	0.949	0.518	1.187
0.05	2.863	0.93	0.07	14.3	1.1553	0.809	2.31	1.01	2.89
0.052	2.98	0.967	0.033	30.32	1.4817	1.039	3.09	1.296	3.86
0.0545	3.065	0.995	0.006	200	2.301	1.61	4.94	2.012	6.16
0.0538	3.08	1.0	0						

By virtue of the form of equation (2) the results for the three aileron settings can now be generalized, and we proceed

to define the effect of the angular velocity - when becoming constant - on the angle of roll. As previously stated, the conditions are identical for all aileron settings, so we cite but one example.

According to Figure 3, the angle of roll is $\mu = 17.82^\circ$ after 1.61 s, that is, after entry of the uniform angular velocity of 0.1931(1/s), with a $\beta = 8^\circ$ aileron setting. Now, supposing the airplane had turned at the outset with a maximum angular velocity $\omega = 0.1941$ without any acceleration; then it would have described an angle $M = 17.9^\circ$. This means, that to disregard the angular acceleration, constitutes an error of the order of $\Delta \cong 0.45\%$, which is very small and becomes still smaller by increasing the time of roll, but greater as the time of roll decreases. It must have attained its maximum as soon as the motion begins; it is, for example,

$$\Delta \mu = \frac{0.366 - 0.0377}{0.366} 100 \approx 90\%$$

after 0.0329 s, although it seldom is a question of time intervals which are much lower than one or two seconds. We can safely ignore any values below that of the period of acceleration and, at that, the error becomes so small that it may be disregarded. Moreover, even a 90% error would hardly become significant, because the period of its validity is very restricted. It is absolutely immaterial whether the distance is covered in 0.03 s or in 0.06 s.

TABLE III

ω	ω^0/s	5.15ω	$1-5.15\omega$	$\frac{1}{1-5.15\omega}$	log	T_1	μ_1	T_2	μ_2
0.02	1.148	0.103	0.897	1.115	0.047	0.0329	0.0377	0.0411	0.0472
0.06	3.44	0.309	0.691	1.45	0.1614	0.1129	0.388	0.141	0.485
0.1	5.73	0.515	0.485	2.065	0.3149	0.22	1.26	0.275	1.576
0.15	8.6	0.772	0.228	4.39	0.6424	0.449	3.85	0.561	4.840
0.16	9.16	0.822	0.178	5.62	0.7497	0.525	4.8	0.655	6.0
0.19	10.9	0.98	0.02	50	1.6989	1.188	12.95	1.483	16.2
0.1931	11.07	0.995	0.005	200	2.301	1.61	17.82	2.012	22.23
0.194	11.11	0.9986	0.001	750	2.875	2.012	22.33		
0.1941	11.12	1.0	0						

TABLE IV

ω	ω^0/s	2.93ω	$1-2.93\omega$	$\frac{1}{1-2.93\omega}$	log	T_1	μ_1	T_2	μ_2
0.02	1.148	0.0586	0.9414	1.062	0.0261	0.01823	0.0209	0.023	0.026
0.06	3.44	0.176	0.824	1.217	0.0853	0.0598	0.205	0.075	0.254
0.1	5.73	0.293	0.707	1.418	0.1516	0.1060	0.608	0.132	0.76
0.15	8.6	0.439	0.561	1.784	0.2514	0.176	1.515	0.22	1.895
0.2	11.46	0.586	0.413	2.422	0.38	4.2	0.269	3.08	0.335
0.3	17.9	0.879	0.121	8.27	0.9175	0.641	11.03	0.801	13.78
0.33	18.9	0.966	0.034	29.43	1.4688	1.027	19.4	1.285	24.3
0.336	19.27	0.985	0.015	66.7	1.824	1.28	24.65	1.55	29.8
0.339	19.45	0.995	0.005	200	2.301	1.61	31.3	2.012	31.1
0.341	19.55	1.0	0						

Now, we try to ascertain whether the uniformity in angular velocity can be attained in the same time interval for other flight speeds.

For $\beta = 8^0$ and $k = 0.02594$, we have

$$T_R = \frac{21}{v} \log \frac{1}{1 - 154.3 \frac{\omega}{v}} .$$

The evaluation is appended in Table V and diagrammatically shown in Figure 6, with the effect of flight speed against the angular velocity and angular acceleration. It is seen that the duration of the angular accelerations diminishes as the flight speed increases.

Table VI records the time intervals up to attainment of constant angular velocity for various flight speeds, along with the angle of roll attained in this interval. It is seen that the same angle of roll is attained for all flight speeds at entry of uniform angular velocity.

The effect of the inertia moment is of a special kind. The inference from (2) is that the angular velocity becomes quickly constant as the inertia moment diminishes and the wing area increases, or more explicitly as $\frac{I}{t\ b^2}$ decreases. This is ascribable to the greater damping in roll, which ostensibly has a material effect as the wing area increases - primarily, as the span becomes greater.

For illustration, we again use the aileron setting $\beta = 8^\circ$. Obviously, the effect of the inertia moment lasts only so long as acceleration prevails; once the uniform angular velocity has made its appearance its role is ended. The duration of the angular acceleration raises with the inertia moment, as Figure 5 shows. After 2.01 seconds, a 25% greater inertia moment reveals the angular velocity constant (Tables II-IV, and Fig. 5) as against $T_R = 1.61$ s with the original moment of inertia. This is a difference of $\Delta T_R = 25\%$. Consequently, the period of the angular acceleration increases in the same measure as the inertia moment.

Upon termination of the acceleration, that is, after 2.01 s, the airplane has executed a turn of 22.23° with a 25% greater

inertia moment, whereas the original angle of roll is 22.33° (Table III). Now the difference amounts to $\Delta\mu = 0.45\%$. This discrepancy diminishes with increasing time or with increasing angle of roll and increases toward $T_R = 0$ or $\mu = 0$. After $T = 0.525$ s, for instance, the difference is

$$\Delta\,\mu = \frac{4.8 - 4.4}{4.8}\,100 = 8.35\%$$

Again disregarding the very short time intervals (perfectly justified in this case also), the inertia moment may be neglected forthwith, and the remaining problem is to prove that a change in inertia moment at small angle and time of roll exerts a greater effect than in the opposite case.

The contention that small inertia moments about the longitudinal axis are preferable for reasons of greater maneuverability, is erroneous, because the effect of this inertia moment on maneuverability is extremely slight. Even under the most adverse conditions, that is, small rate and angle of roll, the effect of a greater inertia moment still remains disappearingly small.

The elimination of inertia moment and angular acceleration makes the calculation extremely simple. For $I\,\frac{d\omega}{dt}$ we deduce $k = \frac{\omega}{v}\,\frac{dc_n}{d\alpha}$ from equation (2), or

$$\omega = v\ k\ :\ \frac{dc_n}{d\alpha} \tag{11}$$

TABLE V

v	$\frac{21}{v}$	ω_{max}	$0.99\omega_{max}$		T	$0.97\omega_{max}$		T	$0.8\omega_{max}$		T
20	1.05	0.1297	0.1284	$154.3 \frac{\omega}{v} = 0.99$	2.1	0.126	$154.3 \frac{\omega}{v}$ 0.97	1.6	0.1038	$154.3\frac{\omega}{v} = 0.8$	0.734
25	0.84	0.1618	0.1602		1.68	0.157	$1-0.97 = 0.03$	1.28	0.1295	$1-0.8 = 0.2$	0.587
30	0.7	0.1941	0.1922	$1-154.3 \frac{\omega}{v} = 0.01$	1.4	0.1885	$\frac{1}{0.03} = 33.3$	1.067	0.1552	$\frac{1}{0.2} = 5$	0.49
35	0.6	0.2263	0.2243	$\frac{1}{0.01} = 100$	1.2	0.22	log 33.3=1.523	0.912	0.1814	log 5=0.699	0.4195
40	0.525	0.259	0.2564		1.05	0.2516		0.799	0.2073		0.367
45	0.466	0.2916	0.2886	log 100 = 2	0.93	0.283		0.71	0.2335		0.326
50	0.42	0.324	0.321		0.84	0.3145		0.639	0.2593		0.284
v			0.7ω			0.5ω		0.2ω	0.2ω		
20			0.0906	$154.3 \frac{\omega}{v} = 0.7$	0.55	0.0648	$154.3 \frac{\omega}{v} = 0.5$	0.316	0.0259	$154.3 \frac{\omega}{v} = 0.2$	0.102
25			0.1132	$1 - 0.7 = 0.3$	0.44	0.0809	$1 - 0.5 = 0.5$	0.253	0.0324	$1 - 0.2 = 0.8$	0.081
30			0.136	$\frac{1}{0.3} = 3.33$	0.366	0.0971	$\frac{1}{0.5} = 2$	0.210	0.039	$\frac{1}{0.8} = 1.25$	0.068
35			0.1587		0.31	0.1132	log 2 = 0.301	0.186	0.045	log 1.25 0.0969	0.058
40			0.1815	log 3.33 = 0.523	0.27	0.129		0.158	0.052		0.051
45			0.204		0.24	0.1458		0.14	0.0583		0.045
50			0.227		0.22	0.162		0.126	0.065		0.041

Now we can compute the angular velocities for any flight speed and aileron setting (Table VII, Figs. 7 and 8).

TABLE VI

v	20	25	30	35	40	45	50
T	1.6	1.28	1.067	0.912	0.799	0.71	0.639
ω	7.22	8.99	10.8	12.61	14.4	16.22	18.03
μ	11.5	11.5	11.5	11.5	11.5	11.5	11.5

TABLE VII

v	$\beta = 2^\circ$ k=0.00719	4° 0.01397	8° 0.02594	12° 0.0366	16° 0.0455	20° 0.0525	
20	0.0359	0.06985	0.1297	0.183	0.2275	0.2625	Angular velocity
25	0.0448	0.087	0.1618	0.2283	0.284	0.3275	
30	0.0538	0.1045	0.1941	0.274	0.341	0.393	
35	0.0626	0.1218	0.2263	0.3195	0.397	0.458	
40	0.0717	0.1395	0.259	0.3655	0.4545	0.524	
45	0.0808	0.1568	0.2916	0.411	0.511	0.59	
50	0.0896	0.1743	0.324	0.457	0.568	0.655	

TABLE VIII

v	v^2	$\beta=2^\circ$	4°	8°	12°	16°	20°	
20	400	26.9	52.2	97	137	170	196.5	Rolling moments
25	625	42	81.5	151.7	214	266	307	
30	900	60.5	117.7	218.2	308	383	442	
35	1225	82.4	160	297	419	520	600	
40	1600	107.8	209	388	547	680	785	
45	2025	136.2	264	491	693	860	993	
50	2500	168.5	326	606	855	1065	1228	

TABLE IX - For $\beta = 2^\circ$

v m/s	ω	$\frac{\pi}{2}\ \omega\ \frac{v}{9}$	$e^{0.16\omega V}$	$e^{-0.16\omega V}$	$\mu^\circ = (\omega\ T)^\circ$	μ arc	T s	T' s
20	0.0359	0.1149	1.122	0.892	26.9	0.47	13.08	26.16
30	0.0538	0.2582	1.293	0.773	39.6	0.69	12.82	25.64
40	0.0717	0.459	1.582	0.6325	50.9	0.885	12.36	24.72
50	0.0896	0.717	2.05	0.488	61.0	1.062	11.86	23.72

v m/s	ω	$\frac{\pi}{2}\,\omega\,\frac{v}{g}$	$e^{0.16\,\omega\,v}$	$e^{-0.16\,\omega\,v}$	μ^o $=(\omega\ T)^o$	μ arc	T s	T' s
			TABLE X - For $\beta = 4^o$					
20	0.06985	0.2234	1.25	0.8	37	0.645	9.24	18.48
30	0.1045	0.502	1.652	0.605	52.8	0.92	8.8	17.6
40	0.1395	0.893	2.44	0.41	66	1.149	8.24	16.48
45	0.1568	1.128	3.09	0.324	71.5	1.243	7.94	15.88
50	0.1743	1.395	4.03	0.248	76	1.323	7.59	15.18
			TABLE XI - For $\beta = 8^o$					
20	0.1297	0.4145	1.511	0.662	48.8	0.85	6.56	13.12
25	0.1618	0.646	1.905	0.525	57.3	1.018	6.3	12.6
30	0.1941	0.9325	2.536	0.395	67	1.165	6	12
35	0.2263	1.268	3.543	0.282	73.9	1.288	5.69	11.3
40	0.259	1.659	5.15	0.1944	97	1.378	5.31	10.6
45	0.2916	2.1	8.13	0.123	83	1.448	4.96	9.92
50	0.324	2.594	13.33	0.075	85.8	1.498	4.61	9.22
			TABLE XII - For $\beta = 12^o$					
20	0.183	0.585	1.795	0.5575	56.2	0.98	5.35	10.7
25	0.2283	0.913	2.49	0.402	66.6	1.16	5.075	10.15
30	0.274	1.315	3.718	0.2695	74.5	1.30	4.74	9.48
40	0.3655	2.34	10.35	0.0966	84.6	1.475	4.03	8.06
50	0.457	3.66	38.9	0.02573	85.7	1.495	3.27	6.54
			TABLE XIII - For $\beta = 16^o$					
20	0.2275	0.727	2.07	0.483	61.5	1.069	4.7	9.4
30	0.341	1.638	5.13	0.195	78.9	1.377	4.03	8.06
40	0.454	2.902	18.2	0.055	87	1.516	3.34	6.68
50	0.568	4.545	93.4	0.01072	89.5	1.56	2.75	5.5
			TABLE XIV - For $\beta = 20^o$					
20	0.2625	0.84	2.315	0.4352	64.6	1.125	4.28	8.56
30	0.393	1.888	6.59	0.1519	81.5	1.42	3.61	7.22
40	0.524	3.35	28.5	0.03515	88	1.538	2.94	5.88
50	0.655	5.24	188.3	0.00531	89.7	1.568	2.39	4.78

Equation (1) yields the rolling moments for these angular velocities

$$\underline{M}_R = 10.76\ c_{m_r}\ v^2$$

This formula is interpreted in Table VIII, and the computed rolling moments graphed in Figure 9.

The time intervals of the turns for a 180° curve were calculated from equation (3) and appended in Tables IX-XIV, while Figure 10 represents the results graphically.

Resume of the Results

1) The time required to attain a uniform angular velocity is the same for any aileron setting; it diminishes as the flight speed and the size of the wings increases and, to an inconsiderable extent, as the inertia moment decreases,

2) The angular acceleration may be disregarded.

3) The error, resulting therefrom, increases as the time of turn decreases; it is the same for any aileron setting.

4) By a certain aileron setting the same angle of roll is attained for all speeds upon termination of the angular acceleration.

5) The period of the angular acceleration raises in the same measure as the inertia moment.

6) A changed inertia moment is markedly more perceptible at small angle and time of roll than in the reverse case, although the effect is negligible.

7) The size of the inertia moment has practically no effect on the time of turn.

8) The moment of róll increases with the aileron setting and the flight speed.

9) The angular velocity is directly proportional to the flight speed and to the coefficient of rolling moment, and inversely proportional to the span and to the gradient of the normal force.

10) The time required to complete a curve is inversely pro-

portional to the aileron setting and to the flight speed.

Now we check the calculation by actual flight test. We record the time of turn, the flight speed and aileron setting for each curve with the aileron setting indicator shown on Figure 11. The clockwork of the recording drum was started electrically from the observer's seat and ran as long as the circuit remained closed. The deflection was transmitted by the aileron connecting strut to the recording pen and recorded on the drum which had a periphery of 226 mm and rotated at the rate of one revolution per 183.8 seconds. By releasing the apparatus for the duration of one curve only we were able to record several curves on one diagram, which otherwise would have been impossible by the low rate of turn of the drum. Aside from slight oscillations called forth by very fast and abrupt aileron deflections, the device proved very successful.

A 180° curve at different flight speeds and aileron deflections revealed 1) the aileron settings at which the curve was entered and abandoned; 2) the time rate from one zero position of the aileron to the other. This time really should show the duration of the curve, but we stopped at continuously increasing periods, because we took the time beginning at the horizontal and ending there, as in the calculation.

This, however, does not mean that at the end of the curve, or in other words, when the aileron setting is zero again, the airplane has regained its level position. For that reason the

actual curve may last longer than the elapsed time between two zero positions recorded in the diagram. Of course, it is impossible to stop exactly from one level position to the next, because it is a comparatively difficult feat to define, after a curve, just when the airplane is level again. In this case we judged the level position by the horizon, which not only is relatively easy but also yielded very satisfactory results.

A curve was measured as follows: Prior to entering a curve the pilot gives the signal for starting the deflection indicator, thus ensuring the actual zero setting, that is, the position of the ailerons at which the airplane is level. It is of primary importance that the pilot execute the aileron deflections as quickly as possible because the calculation does not make any allowances for time consumed in making the deflection.

The device had to be kept running after the curve was completed to ensure the actual zero position. Then the circuit was interrupted and the pilot executed several abrupt aileron deflections, which in the diagrams appear as simple lines, by means of which it is possible to quickly distinguish the individual curves. In this manner as many as eight curves could be embodied in one diagram.

The charts show that the zero line drawn on the ground does not correspond to that of the test flight, which proves that there is a continuous aileron deflection in level, straight-away flight. The device was mounted on the starboard strut, so

that a deflection upward denotes the initiation of a roll to starboard; the airplane inclines to port. The mean aileron deflection required to neutralize this inclination amounted to approximately 1.5°.

The diagrams of Figures 12 to 15 are interpreted in Tables XV and XVI.

No.		Curve	Aileron setting in °	Average in °	Time according to diagram s	Time measured s	Time calculated s
			TABLE XV				
			Diagram 12. v = 120.7 km/h = 33.5 m/s				
1	starboard	in	4.7	6.9	11	12.8	12.7
		out	9.0				
2	port	in	3.6	6.4	11.4	12.9	13.2
		out	9.2				
3	starboard	in	4.7	6.6	10.6	12.1	12.9
		out	8.4				
4	port	in	3.4	6.5	11.4	13.2	13.0
		out	9.6				
			Diagram 13. v = 120-127 km/h = 34.4 m/s				
5	starboard	in	3.8	6.6	9.3	9.7	12.8
		out	9.3				
6	starboard	in	4.7	6.9	11.4	11.5	12.4
		out	9.0				
7	port	in	6	8.8	11.4	9.8	10.8
		out	11.6				
8	port	in	3.5	5.8	11.7	12.1	13.9
		out	8.0				
			Diagram 14. v = 143 km/h = 39.7 m/s				
9	starboard	in	8.4	9.5	8.5	9	9.5
		out	10.6				
10	starboard	in	5.5	6.1	11	12.2	12.7
		out	6.7				
11	port	in	7.8	9.7	10.2	10.4	9.3
		out	11.5				
12	port	in	4.5	6.5	11.7	10.5	12.2
		out	8.4				

TABLE XVI

No.		Curve	Aileron setting in °	Average in °	Time according to diagram s	Time measured s	Time calculated s
	Diagram 14. v = 127-129.5 km/h = 35.6 m/s						
13	starboard	in out	8 11	9.5	9.4	9.9	10.2
14	starboard	in out	5.4 7.6	6.5	11.3	11.1	12.7
15	port	in out	7.2 10.7	9	8.9	9.2	10.5
16	port	in	3.8 10	6.9	11.4	11.9	12.3
	Diagram 15. v = 143 km/h = 39.7 m/s						
17	starboard	in out	11.1 11.4	11.3	8.3	8.5	8.4
18	port	in out	7.6 12.3	10	8.8	9	9.1

TABLE XVII

Curve	1	2	3	4	5	6	7	8
Time measured s	12.8	12.9	12.1	13.2	9.7	11.5	11.4	12.1
Time computed s	12.7	13.2	12.9	13	12.8	12.4	10.8	13.9
Difference %	0.8	2.3	6.2	1.5	24.2	7.3	5.6	13

TABLE XVII (cont.)

Curve	9	10	11	12	13	14	15	16	17	18
Time measured s	9.0	12.2	10.4	11.7	9.9	11.3	9.2	11.9	8.5	9
Time computed s	9.5	12.7	9.3	12.2	10.2	12.7	10.5	12.3	8.4	9.1
Difference %	5.3	3.9	11.8	4.1	3	11	12.4	3.3	1.2	1.1

Table XVII incorporates the differences between the figures arrived at by calculation and in actual flight tests in per cent. Apart from curve 5, with a discrepancy of 24.2%, the agreement averages very satisfactorily.

The theoretical figures are slightly higher than those of the test flight, which is due to the fact that the calculation makes no allowance for the resultant of the lifting forces which in inclined and in curve flight is no longer in the axis of symmetry but slightly outside of it. This aids the work of the ailerons; the airplane rolls more rapidly. The average discrepancy is 6.6% for all 18 curves, according to Figure 18.

A subsequent check by the method suggested by Kann and Salkowski yielded less satisfactory results. The discrepancy here amounted to nearly 8%, but it should be remembered that steady circling flight, particularly at higher altitudes, is not very easy.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

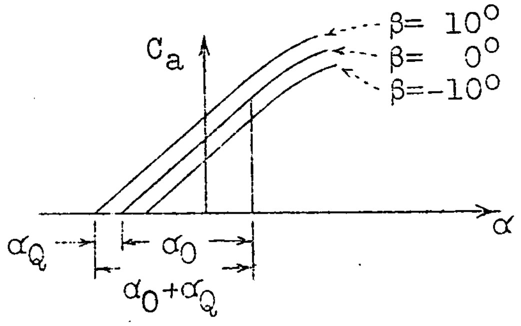


Fig. 1

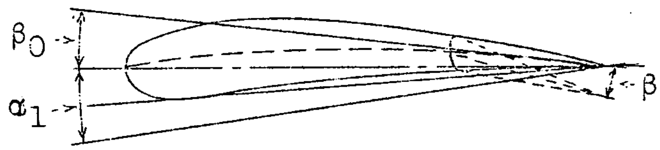


Fig. 2

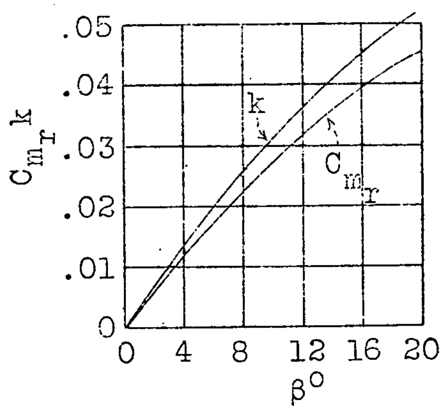
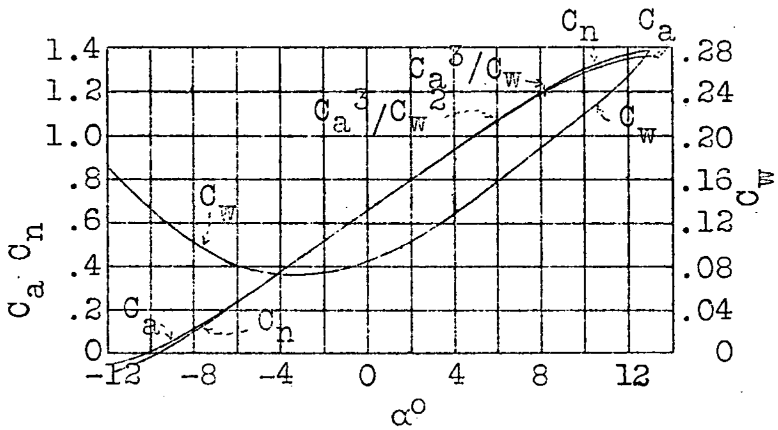
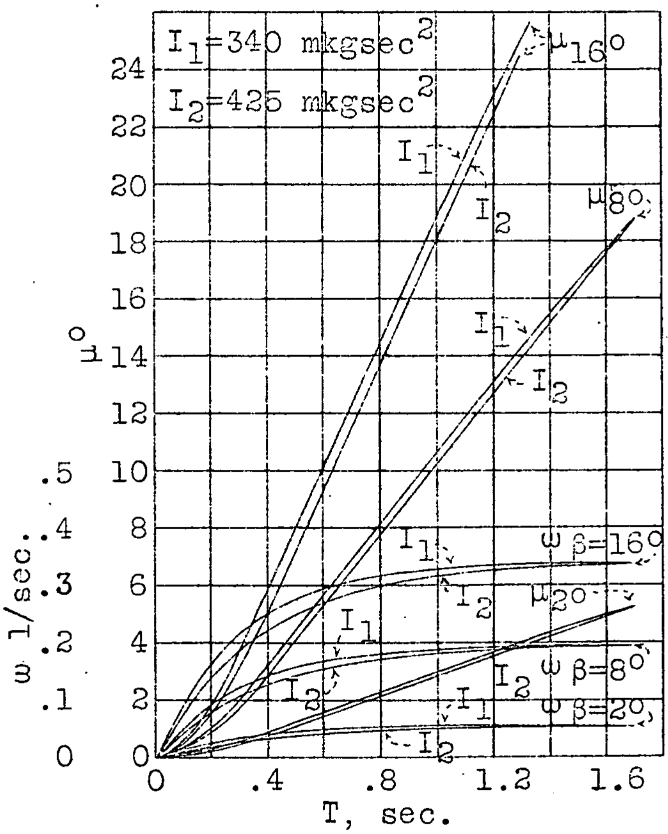


Fig. 3



C_n
C_a
$C_a{}^3/C_w{}^2$
$C_a{}^3/C_w$
C_w
C_a
C_n
$C_a \; C_n$
C_w
1.4
1.2
1.0
.8
.6
.4
.2
0
.28
.24
.20
.16
.12
.08
.04
0
-12
-8
-4
0
4
8
12
α^o

Fig. 4



I_1=340 mkgsec^2
I_2=425 mkgsec^2
μ_{16^o}
μ_{8^o}
μ_{2^o}
$\omega \; \beta=16^o$
$\omega \; \beta=8^o$
$\omega \; \beta=2^o$
I_1
I_2
μ^o
ω 1/sec.
T, sec.
0
.4
.8
1.2
1.6

Fig. 5

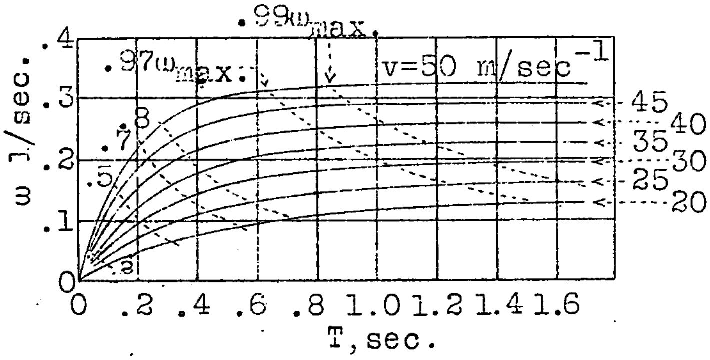


Fig. 6

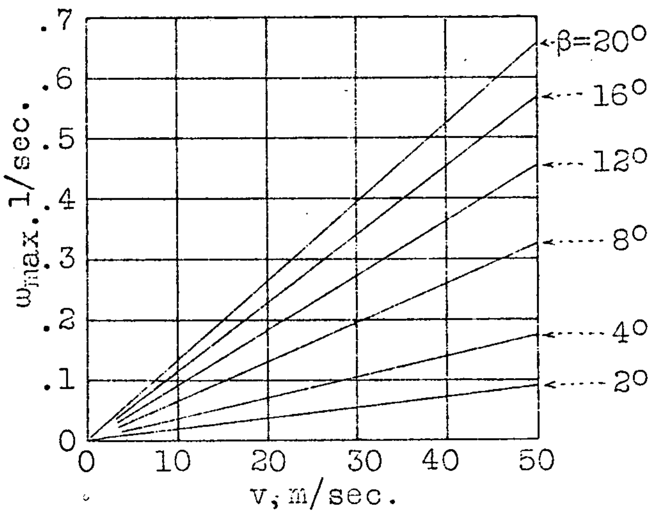


Fig. 7

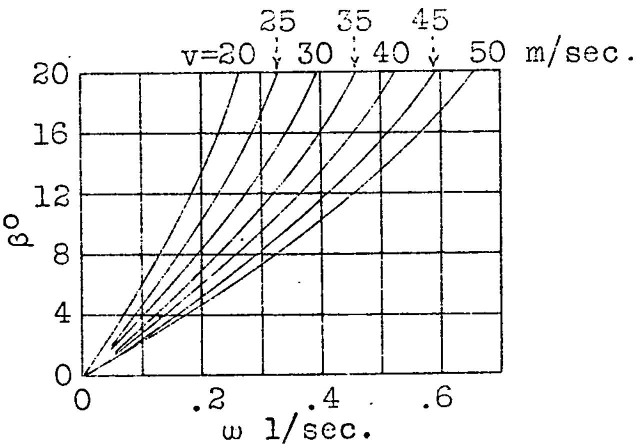


Fig. 8

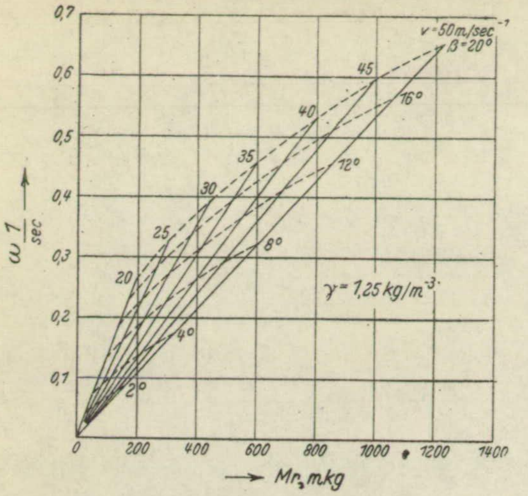


Fig. 9

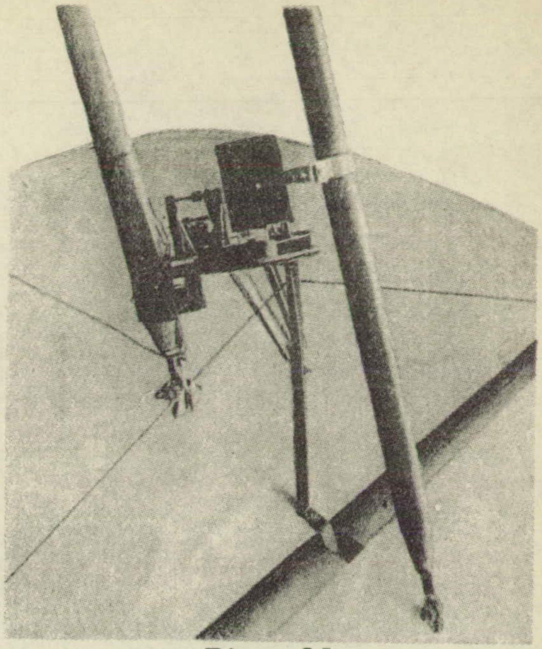


Fig. 11

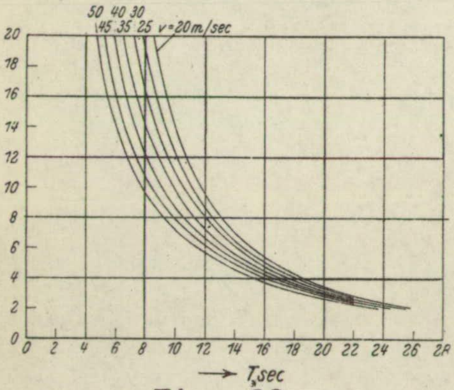
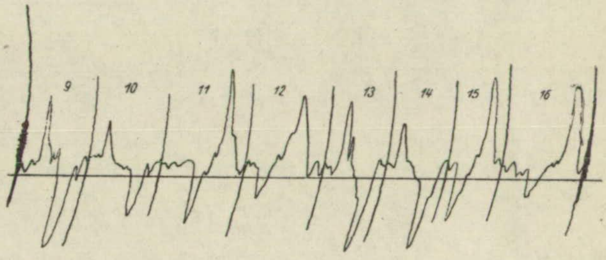
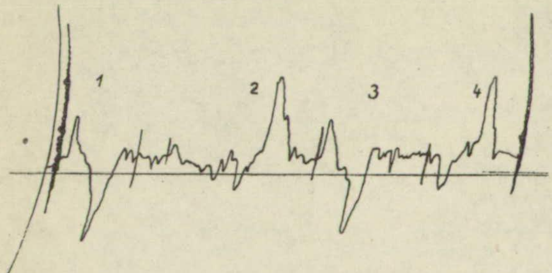


Fig. 10



Beseler Kruse
6.III.30 600 m

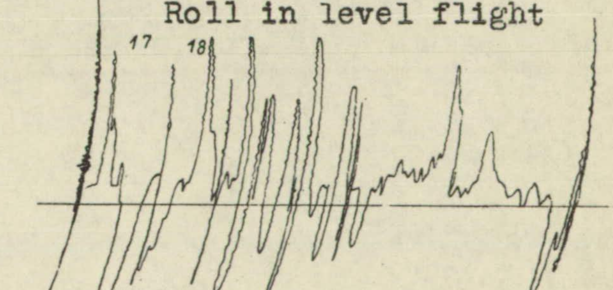
Fig. 14



Beseler, Kruse 5 III 1930
H D35
600 m No 324

Fig. 12

Roll in level flight



Beseler Kruse
6 III.30. 600 m

Fig. 15

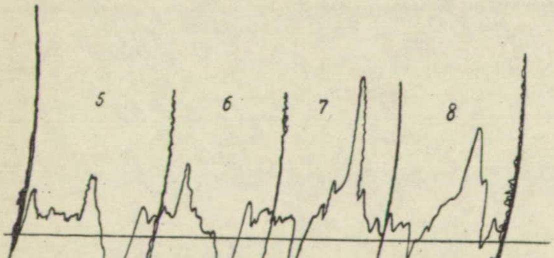


Fig. 13